Conference on Undergraduate Research in Mathematics

Penn State University
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Talk Abstracts

Plenary Speakers

George Andrews, Penn State University

The Indian Genius, Ramanujan: His Life and His Incredible Mathematics

This talk focuses on the famous Indian genius, Ramanujan, and will provide an account of his amazing, albeit short life. We shall try to lead gently from some simple problems involving Fibonacci numbers to a discussion of some of Ramanujan's mathematical discoveries. We shall focus on matters related to continued fractions.

Frank Morgan, Williams College

Surfaces with Density and the Poincaré Conjecture

Calculus teachers and physicists have long studied surfaces with density (which you integrate to get mass). There has been a recent surge of interest in the geometry of such surfaces, ranging from probability to Perelman's recent proof of the Poincaré Conjecture to recent advances by undergraduates.

Student Speakers

Vivek Agarwala, Phillips Exeter Academy

An Analysis of Bounded Diagonally Non-Recursive Functions

The author researched the (Turing) degrees of uncomputability of diagonally non- recursive (DNR) functions bounded by constant, linear, quadratic, and other power function bounds. The main effort was towards determining if the Turing degrees of DNR functions coincide for various power function bounds. While it was proven that DNR functions bounded by power functions of order > 1 do not coincide with those that are constant bounded, the questions regarding linear bounds remain open. However, the author believes that this paper elucidates new knowledge about this class of functions.

Haris Amin, Wabash College

Simulation of Viscoelastic Fluids

Viscoelastic fluids have properties of both solids and fluids, and as a consequence, are difficult to simulate on a computer. In this research, we simulate a viscoelastic fluid passing from a large channel to a small channel. The numerical method we use is validated by analyzing the dependence of corner vortex reattachment lengths on the viscoelastic nature of the fluid.

We employ a weighted-norm least-squares finite element method to approximate the solution to the Oldroyd-B equations. The viscoelastic nature of the fluid is characterized by the Weissenberg number, indicating the balance between the fluid and solid character of the fluid. Large Weissenberg number simulations are considered one of the most challenging problems in numerical analysis. Here, we investigate the appropriate weighted norms and approximation spaces to use. Our numerical tests confirm vortex reattachment lengths comparable to benchmark solutions. We've also found that the use of quadratic basis functions results in the most efficient process to do this. Flow through domains with reentrant corners results in solutions with singularities, indicating unnatural infinities in the solution. Our tests indicate that one possible shortcoming of many methods is the inability to directly treat these corner singularities. Our method has a natural way to deal with these difficulties and our results are in agreement with accepted values.

Amine Benkiran, Shepherd University

Harvesting Natural Resources

In this talk, we present a differential equation model for the population of salmon with no harvesting, constant harvesting and proportional harvesting schemes. First, the model is solved analytically for the population P(t). The goal of this project is to decide on a good harvesting scheme which will lead to a good catch and at the same time does no deplete the salmon population.

Krystin Benkowski, Gannon University

Low Genus Actions on Riemann Surfaces

This work was part of an 8 week NSF funded REU program at the University of Portland. We examined the actions of low genus groups on Riemann surfaces. We classified the surfaces those groups acted on according to the genus of the surface. The Riemann-Hurwitz formula was used extensively.

April Bowers, Houghton College

Tiling 3 x n rectangles with trominoes and 4 x n rectangles with tetrominoes

Recently a solution to finding the number of tilings for 3 x n rectangles with triominoes was solved. The solution technique employed does not generalize readily to the problem of tiling 4 x n rectangles with tetrominoes. In this note, we develop another approach to the triominoe problem which will generalize to the tetrominoe problem. Employing this technique we discuss the two problems of finding the number of tilings for a 4 x n rectangle with T and L tetrominoes and also with T, L, and Z tetrominoes.

Jared W. Burns, University of Pittsburgh, Greensburg

The Integrability of 1-Forms on R^3

This presentation will discuss the integrability of Pfaffian forms on R^3. We will give the definition of closed and exact forms. Our main objective is to find the necessary and sufficient conditions for a one-form to be exact up to a multiple. We will demonstrate how some of the conditions can be computed using MAPLE. Examples will be provided.

Tyler Drombosky, Youngstown State University

Effective Condition Number

The condition number, k(A), is very useful when determining the accuracy of solutions to linear systems when using numeric solvers. However, recent meshless techniques for approximating partial differential equations have been known to create ill-conditioned matrices, yet are still able to produce results that are close to machine accuracy. We consider the relationship between the effective condition number, k_{eff}(A,b), and the accuracy of approximations for ill-conditioned linear systems that arise when using the Method of Fundamental Solutions.

Daniel Esslinger, Penn State University

Geometric Connell Sequences

In 1959, Connell introduced the sequence 1, 2, 4, 5, 7, 9, 10, 12, 14, 16, ... in the problem section of the American Mathematical Monthly. Many people subsequently studied this sequence and its many properties and along the way generalized it in a number of directions. In this talk, I will generalize the Connell sequence in a completely new direction by considering geometric rather than arithmetic sequences. This will involve a two-parameter generalization using a modulus m and a base q. I will close with an investigation of the meaning of the last numbers in the corresponding subsequences.

Eddie Feeley, Seattle University

Stability Analysis of Solutions to the Simplified Trulsen and Dysthe Equation

The simplified Trulsen and Dysthe equation (rA) is a generalization of the nonlinear Schrödinger equation and can be used to model the evolution of surface waves on deep water. We present numerically determined nontrivial-phase solutions to rA and determine their stability.

Alison Green, Canisius College

A Cost Analysis of the Human Papillomavirus: Individual Education vs. Mass-Media Campaign

Human Papillomavirus (HPV), a common sexually transmitted virus, is a collection of more than 100 viruses, some of which (called "high-risk" oncogenic or carcinogenic HPV) are associated with certain types of cancer. HPV 16 and 18 cause approximately 70% of all cervical cancer. An estimated 11,150 new cases of cervical cancer will develop in the U.S. in 2007. In 2006, the FDA approved the first vaccine to prevent cervical cancer, designated for females ages 9-26, called Gardasil. Studies have indicated Gardasil has a 95-100% success against HPV types 6, 11, 16, and 18. Many studies have shown that there exists a lack of HPV awareness, as well as knowledge of causes, effects and preventive measures. In this study we compare two strategies for controlling the spread of carcinogenic HPV in the population, while minimizing cost. The research is conducted via a cost analysis of a mandatory vaccination policy vs. a mass-media awareness campaign, each individually modeled with a system of differential equations. Mandatory (but not universal) vaccination includes individual-based education at the time of vaccination only, while the mass-media campaign is assumed to be ongoing. In both cases the education influences females to get vaccinated and/or reduce their sexual activity, but is of limited duration. We use qualitative analysis to derive the respective control reproductive numbers, and numerical analysis to obtain the total costs of vaccination, education, high sexual activity, and expected cancer treatment costs for infected females in both models. Our results support the conclusions of a 2005 study analyzing the epidemiology of HPV with a potential vaccination that, even in the presence of a vaccine, the infective population will remain large due to a high transmission rate. Our results also support the conclusion that a high transmission rate and a high reproductive rate require a high efficacy and high vaccine coverage to eliminate the epidemic.

James Greene, Penn State University

Scavenger Models and Chaos

In the talk we discuss 2 basic models for a three species scavenger system. The simpler model is discussed first. We begin by performing the basic linear analysis of fixed points for the cases when an interior fixed point exists. Then we analyze the system for varying parameter values, particularly looking for when and where Hopf bifurcations occur. We fix all but 2 parameters and vary these, analyzing bifurcation diagrams for different types of behavior. As one varies parameter values, periodic orbits begin to undergo period doubling, eventually cascading into a strange attractor. We analyze this strange attractor using Poincare sections. Lastly, we briefly analyze a more complicated scavenger model, which has much more complicated dynamics.

Jarod Hart, University of Wisconsin, La Crosse

Inflation Rates under Uncertainty: A Statistical Approach

We study a game theoretic economic model involving symmetric agents who compete by bidding to purchase goods in a series of stages. The production of the good is assumed to be from an unknown distribution. Each agent bids according to the Nash equilibrium strategy that incorporates the history of observed production. As more information is obtained, the agents learn more about the unknown distribution by way of Bayesian learning. We compare the expected inflation rate for each stage to the inflation rate when the production distribution is known, which is established by the Fisher equation. When the production distributions is unknown, the expected inflation deviates from results of the Fisher equation. Although as more is learned about the unknown distribution over time, the expected inflation converges to the inflation rate determined by the Fisher equation.

Emily Hendrickson, Slippery Rock University

Stabilizing the Vibrations of a Thermoelastic Beam

The vibrations of a one dimensional thermoelastic beam are modeled by a system of partial differential equations. It has recently been shown that the vibrations of the beam can be uniformly stabilized using the thermal properties of the beam or by applying mechanical damping to its boundary. The purpose of this talk is to provide a mathematical comparison of the effectiveness of the two different forms of damping. This work provides the foundation for future research involving thermoelastic plates and shells with numerous applications in aerospace engineering.

Leland J. Jefferis, Seattle University

Stability of nontrivial-phase solutions of the nonlinear Schrödinger equation

The nonlinear Schrödinger equation is a two-dimensional nonlinear partial differential equation that can be used to model a wide range of physical phenomena including the evolution of surface waves on deep water and pulse propagation in optical fibers. NLS admits a class of traveling-wave solutions with nontrivial phase. We study the stability of these solutions asymptotically and numerically in order to establish that they are unstable.

Stefan Carl Johnson, California State University, San Bernardino

Long Term Behavior of Solutions for Riccati Equations

The Riccati equation has been studied since the early 1700s, and in this time, mathematicians have developed different methods of solving its special cases. This talk will focus on initial value problems where the Riccati Equation has constant coefficients. We will derive necessary and sufficient conditions for solutions to stay bounded, as well as conditions for unbounded growth in finite time (blow-up).

Courtney Jones, Penn State University

Force-Dipole Model of a Self-Propelled Bacterium at low Reynolds Number

Different species of bacteria have different methods of moving through fluid. In this study, we consider Monotrichous bacteria, which have a single flagellum. The flagellum is located on the tail end of the body and is composed of contractile elements. The bacteria use their flagellum to propel themselves head first through the surrounding fluid. We are interested in the hydrodynamic interactions between a bacterium's velocity and an otherwise stationary fluid. The bacteria's only interaction is through the fluid. Therefore, to analyze the hydrodynamic interactions of the fluid with a self-propelled bacterium, as the later one generates and reacts to the fluid's local velocity, we can consider Navier-Stokes equations. Linearizing Navier-Stokes equation, because of the low Reynolds number, yields the linear Stokes equation. Based on Stokes equation and the dynamics of a bacterium, we compute the disturbance due to self-propelled bacteria. The benefits of the force-dipope model of bacteria will be presented at the conclusion of the pesentation.

Derrick Keister, Penn State University

Linear Algebra and a Product of Pell Numbers

In this talk, I will prove two Pell Number identities by the use of linear algebra concepts. After first some history of Pell numbers and their similarities to the Fibonacci numbers, I will prove an identity that mimics that of an already known Fibonacci identity. In conclusion, I will use this identity to prove another, more nontrivial, Pell number identity. These two proofs will conclude my talk.

J. Zachary Klingensmith, Washington and Jefferson College

Van Roomen's Equation: Solving a 45th Degree Polynomial Equation

Students today are taught to solve basic polynomials, mainly of degree two and three, using algebraic methods such as factoring. In an interesting historical problem, Francois Viète looked at a 45th degree polynomial and immediately came up with one solution, shortly followed by the other 22 recognized at the time. I will show Viète's procedure for solving this equation using trigonometry and a second, more modern approach, unavailable to Viète.

Natalie Komarov, Carnegie Mellon University

Normal Subgroups of the Free Group

We present a method of producing normal subgroups of F(r), the free group on r generators, not containing a given word. Special attention is given to the normal subgroups of the F(2). We use connections with algebraic topology to produce such subgroups and clearly state for what class of words these subgroups are not easily obtainable.

Chor Hang Lam, Dartmouth College

Matrix Minors and Chamber Clusters

In understanding totally nonnegative matrix, we will express matrix minor in terms of some specified collections of minors of the same matrix using double wiring diagrams. One of the problems is to show that these expressions are Laurent and subtraction-free. We also extend this to non-square matrix where Subtraction-Free Laurent expressions are found for the 2 by n case. Another meaningful simplification is the single wiring diagrams and their associated graph; we will discuss the diameter, levels and other combinatorial results of the graph and how it relates to the properties of the double wiring diagrams.

Benjamin Leard, James Madison University

Dynamics of a Ratio-Dependent Predator-Prey Model with Nonconstant Harvesting Policies

Predator-Prey models have been used in ecology, biology and economics to understand and predict the behavior of predator/prey interactions. We analyze a Ratio-Dependent Predator-Prey model that involves harvesting on the prey population. Our findings include calculating a maximum sustainable yield and detecting multiple bifurcations and connecting orbits.

Rebecca Lyzinski, St. Mary's College of Maryland

Exploring a Surplus Production model for Pacific Halibut

The Pacific halibut (Hippoglossus stenolepis) provide a commercial fishery worth close to one and a half billion dollars a year. The large flatfish, weighing up to 500 pounds, is regulated by the International Pacific Halibut Commission (IPHC) for six regions of the Pacific Ocean. The IPHC gathers data on commercial harvest and effort (the number of hooks used) each year. Using 29 years of data, we modeled Pacific halibut populations using a difference-equation based surplus production model to predict commercial harvest. Four parameters (initial biomass, carrying capacity, growth rate, and catchability) were estimated to achieve the best fitting

model using maximum likelihood techniques. Initial biomass and carrying capacity were estimated for each region. Different models were created using combinations of one or six growth rates and one or two catchabilities. The six growth rates represented one growth rate for each region and the two catchabilities represent one for pre-1983 and one for post-1983 data to account for a change in fishing gear in 1983. An additional model was created using a parameter to account for density dependent catchability. Akaike's Information Criterion was used to weigh goodness of fit against complexity to determine which model fit the best.

Lexie Martin, University of Southern Mississippi

On the Volume of a Tetrahedron

We proved a new volume formula of a tetrahedron in terms of the angles of the edges from one vertex of the tetrahedron, using linear algebra.

Vincent Martinez, The College of New Jersey

Convexity in the Space of Real, Compact Sets

In n-dimensional Euclidean space, the notion of convexity is very simple. A set C is said to be convex if for each pair of points x,y in C, the Euclidean line segment joining x,y is also contained in C. How could one extend this notion to a more general metric space, in particular, to the space equipped with the Hausdorff metric of nonempty, compact subsets of Euclidean space? This talk will present such an extension and some interesting consequences of it.

Joe Pleso, Penn State Erie

Surreal Analysis of Go

Go is a strategic board game for two players. Surreal numbers, a superset of the real numbers, can describe positions found in Go. Sometimes, the outcome of a small section of the board is independent of the remainder of the board. Methods for analyzing these

“rooms” include the use of software that has application to artificial intelligence, and will be the primary focus of the talk. The level of this talk is suitable for a general audience.

Douglas Rizzolo, Harvey Mudd College

Searching for fixed points in infinite dimensions

Fixed point theory, the theory of when a function has a point that is mapped to itself, is a particularly elegant field of mathematics that has found wide ranging applications including in economics and differential equations. However, many fixed point theorems are proved with non-constructive approaches. Hence, while they guarantee the existence of a fixed point, they provide little insight into how that fixed point can be found; this can be frustrating in application. For example, in differential equations, fixed point theorems can be used to guarantee the existence of solutions to broad classes of differential equations, but often do not provide any information beyond existence. In this talk I will outline a semi-constructive proof of Schauder's fixed point theorem and, time permitting, will give an example of how this proof can be turned into a method for approximating solutions to differential equations.

Jared Ruiz, Youngstown State University

Building on Babel

It's a question everyone asks: given a finite field, find an element of high order. The solution, however, is often not so trivial. By using an infinite tower of finite field extensions, we can generate a sequence of elements whose degree and order grows with every extension.

Matthew Russell, Taylor University

Investigating p-colorability of Twisted Torus Knots: Methods and Results

This is the second of two talks that report on the p-colorability of several families of T(m,n,r,s) twisted torus knots. We present theorems for calculating the p-colorability of the families of T(m, n, r, s) twisted torus knots for n congruent to ±1 (mod m) by finding their determinants. Instead of the usual method of reducing crossing matrices to find the determinant, we describe a new method that is applicable for braid representations with full cycles and twists. The techniques that we will use are original but fairly elementary, requiring only a little bit of linear algebra.

Jonathan Schrock, Taylor University

Colorability and Determinants of T(m,n,r,s) Twisted Torus Knots for n congruent to ±1 (mod m)

This is the first of two talks that report on the p-colorability of several families of T(m,n,r,s) twisted torus knots. Our research was motivated by results obtained by another undergraduate research team on the p-colorability of torus knots. This talk will describe some of the mathematical background needed to understand our research. Also, it will give an overview of how we developed our initial conjectures on the behavior of the knot determinants from a computer program we developed for this project.

Derek Seiple, University of Arizona

"Mom! There's an Astroid in my Closet!"

We address the mathematics of closet doors. Specifically we answer the questions, "How much clear floor space is required to open and close a bifold closet door, and what is the boundary curve?" We then generalize the problem to address n-fold doors and show a connection to Archimedes' construction of an ellipse using the Trammel of Archimedes.

Peter Schallot, Slippery Rock University

A Generalization of Nash’s Equilibrium Theorem

We show that any game having compact, convex strategy sets and affine, continuous payoff functions has an equilibrium point. Our proof utilizes Sperner's Lemma, the Knaster-Kuratowski-Mazurkiewicz Theorem, and Kulpa's Indexed Family Theorem.

Mark Spindler, Carnegie Mellon University

Edge-Disjoint Paths in the Directed n-Cube

In the directed n-cube, given a downward-closed set A, and an upward-closed set B, let p(A,B) be the maximum number of edge disjoint paths from A to B. We are concerned with finding the minimum of p(A,B) across all pairs of sets A,B of fixed sizes a,b. This problem is a generalization of a conjecture posed by Bollobas and Leader. We present some preliminary results which help in analyzing the problem.

Tracy Stepien, University at Buffalo

A Mathematical Model for Idiopathic Intracranial Hypertension

Idiopathic Intracranial Hypertension (IIH) is a syndrome of elevated intracranial pressure due to an unknown cause. Some evidence suggests than an overly collapsible sinus combined with an increase in cerebral blood flow could be the cause of IIH. A mathematical model of intracranial pressure dynamics is developed to investigate the etiology of IIH. The model is based on Marmarou's electrical circuit model which was found to be fairly reliable even compared to a more complex model such as Ursino's. A collapsible sinus is modeled by a downstream Starling-like resistor. The model is validated by comparing simulations to clinical data. It is demonstrated that a stable elevated pressure state can be introduced via a saddle-node bifurcation in the parameter domain. These results support the theory that an abnormally collapsible sinus and/or elevation of cerebral blood flow are sufficient to explain the intracranial hypertension observed in IIH.

Kye Taylor, University of Colorado at Boulder

Denoising Manifold-valued data with Eigenfunctions of the Laplace-Beltrami Operator

In cases where a low-dimensional manifold is embedded in a higher-dimensional space, manifold learning is complicated by noisy measurements and bad sampling. To improve the estimation of the embedded manifold, and thus obtain a better parameterization of the dataset, we construct a low-pass filter to remove noise using approximations to the eigenfunctions of the Laplace-Beltrami operator as defined on the manifold.
We estimate this operator by constructing a graph to serve as a discrete approximation to the manifold, and thereby numerically obtain a basis set of harmonic functions that can be regarded as a Fourier basis for L2 functions defined on the manifold. So not only are we able to uncover the manifold, but we can also obtain a better parameterization of functions whose support is the manifold. This approach to denoising the dataset is analogous to solving the heat equation on the manifold with an initial temperature profile given by the original dataset. As the temperature solution evolves, high-frequency components comprising the initial distribution are eliminated. For short times into the evolution of the temperature, the low-pass filter accomplishes the same task.

Deborah Vicinsky, Bucknell University

The Optimality of Solutions to the Stable Marriage Problem and its Variations

The classical stable marriage problem involves a community of n men and n women, each of whom ranks the members of the opposite sex in order of preference. A matching is considered stable when all of the men and women are paired so that there are no man and woman who are not partners who both prefer each other to their actual partners under the matching. In this talk, we will describe the number of stable matchings possible for certain pairs of preference matrices and what it means for a matching to be optimal. Finally, we will explore instances of the stable marriage problem in which there are different numbers of men and women, if ties are allowed, and if the men and women are allowed to declare members of the opposite sex to be unacceptable.

Douglas Wajda, Youngstown State University

Differential Equations of Freefall and Skydiving

A look into the differential equations that govern the motion of a person in free-fall and also with a parachute deployed, ultimately leading to an answer to how long a person could wait to pull their parachute and still land safely on the ground.

Jonathan Wherry, Oregon State University

Population Models and the Implications of Local Stability

Local stability implies global stability in many of the "standard" population models, however this is not always the case. Sometimes it is possible to have a locally but not globally stable model. This talk will focus on some simple modifications that can be done to some globally stable models such that local stability still implies global stability. It will always mention various ways of seeing how instabilities in models may occur.

Kyle White, Penn State Erie

Adding a Random Walk in Carrying Capacity to a Surplus Production Model

Pacific halibut are large flatfish weighing up to 500 pounds and a highly important commercial fishing industry. Currently the International Pacific Halibut Commission (IPHC) regulates halibut fishing levels in 6 regions of the Northern Pacific Ocean. The IPHC collected 29 years of fishing data in an attempt to model and regulate halibut populations. We considered a difference equation model to predict commercial harvest based on biomass surplus production. The model initially contained four parameters for each of the six regions: initial biomass, the growth rate, the carrying capacity, and the catchability (how effective the gear is for catching halibut). Maximum likelihood techniques were used to fit each of the 6 regions simultaneously, comparing observed and predicted harvests. At first the model fit was adequate, but we felt it could be improved by relaxing some assumptions of the model. Specifically, the carrying capacity was assumed to be constant whereas it likely varies over time. To compensate, the carrying capacity was modeled with a bounded random walk from year to year. This improved model fit and allowed us to find trends in the carrying capacity. Markov chain Monte Carlo methods were used to determine prediction intervals for future halibut biomass levels.

Lucas Willis, Rowan University

Euler and Oblique Angled Diameters

In his paper on the properties of conic sections (E83), Euler demonstrates a proof that if a curve has two parallel oblique angled diameters, then it must have infinitely many. We will also examine his determination that all appropriately defined diameters of the conic sections are oblique angled diameters. The notion of an oblique angled diameter is likely foreign to most modern mathematicians; however, we will define them and talk about their properties.

Aaron Yeager, Missouri State University

Characterization of the Vertex-Reinforced Random Walk and Trapping Subgraphs

The vertex-reinforced random walk (VRRW) is a random walk on a graph G which is more likely to visit vertices it has visited before. In the long run, the VRRW restricts itself to a finite set of vertices called a trapping subgraph. This behavior of the VRRW differs from that of the usual random walk, which, with probability one, will not be bounded to a finite set except on finite graphs. In this paper we investigate how to find a trapping subgraph and the process by which the VRRW settles down in to one of these subgraphs. We also describe the probability distribution of the VRRW on these trapping subgraphs.

Peng Zhao, Cornell University

A Webpage Reputation Scoring System

As the Internet continues to grow, it is also rapidly becoming a more dangerous place. The idea of webpages downloading malicious softwares to your computer or phishing webpages trying to steal your credit card information is no longer foreign. In this research, we looked into giving each webpage a "reputation score" based on data collected by crawlers, such as spywares, phishing and SSL certificates. We proposed several scoring models using methods from Statistical Learning Theory. I will talk about the advantages and shortcomings of each model and the results we obtained. The problem became more interesting and complicated when the webpage data were collected over time. Distinguishing between Hacker and Hacked pages was a key issue. We discovered that the theory of

Hidden Markov Models could be applied to answering this question. I will illustrate this connection in my talk. This project was sponsored by the Research in Industrial Projects for Students (RIPS) program at UCLA and Symantec Corporation.