Conference on Undergraduate Research in Mathematics

Penn State University
November 20-21, 2009

Talk Abstracts

Plenary Speakers

Art Benjamin, Harvey Mudd College

Combinatorial Trigonometry

Many trigonometric identities, including the Pythagorean Theorem, have combinatorial proofs. Furthermore, some combinatorial problems have trigonometric solutions. All of these problems can be reduced to alternating sums, and are attacked by a technique we call D.I.E. (Description, Involution, Exception). This technique offers new insights to identities involving binomial coefficients, Fibonacci numbers, derangements, zig-zag permutations, and Chebyshev polynomials.

Aparna Higgins, University of Dayton

Demonic Graphs and Undergraduate Research

Working with undergraduates on mathematical research has been one of the most satisfying aspects of my professional life. This talk will highlight some of the beautiful and interesting research done by my former undergraduate students on line graphs and pebbling on graphs. We will consider line graphs, some pioneering results in pebbling graphs, and pebbling numbers of line graphs. This work has inspired other students to investigate questions in these areas, and it has contributed to my research as well.

Student Speakers

Austin Anuta-Darling, West Virginia University

Signatures of Chaos in Traveling-Wave Electrophoresis

Traveling-wave electrophoresis (TWE) is a method of ion separation and mobilization which utilizes longitudinal electric field waves traveling through a stationary fluid medium. In this research a 2-dimensional computer model of TWE using a trigonometric electric potential is studied in order to determine the existence and properties of chaos which occurs in particle motions. The primary mathematical tools used to analyze this chaos include: Lyapunov exponents, correlation dimensions, particle displacement maps, and average particle velocity graphs. These tools demonstrate the existence of particle motions which can be either periodic or chaotic. The average velocity graphs are found to contain devil's staircase structures that reveal a correspondence between map winding numbers and average velocities. This correspondence implies the existence of quasi-periodic states which are neither periodic nor chaotic.

Myles Baker, Baylor University

Modeling Financial Derivatives: An Analysis of Uniform and Nonuniform Schemes For Numerical Solutions to the Black-Scholes Equation

The Black-Scholes equation can be used to model option pricing, but the volatility of financial markets create irregularities that cause the model to be highly inaccurate. Investment reliability is crucial to establishing confidence in financial markets, and Black-Scholes analysis can be applied to create directionality models and calculate minimum-risk investments, which generally yield greater activity and increased market values with less liability. In this paper we show that numerically stable explicit, implicit and leapfrog finite difference schemes can be used to evaluate the Black-Scholes partial differential equation without some of the limitations characteristic of traditional numerical methods. Solutions and simulations on nonuniform grids are of particular interest to us because of potential applications, and their relationship to uniform grids will be analyzed for comparative purposes. We utilize Taylor expansions to evaluate the consistency and accuracy of the finite difference schemes along nonuniform grids based on different choices of temporal and spacial derivative approximations. Precise truncation error terms are derived and analyzed, and computer simulations using Matlab software are given to illustrate their efficacy. We show that, though orders of accuracy are lower compared with their peers on uniform grids, new numerical methods to solving the Black-Scholes partial differential equation possess significant potential in computing unsteady or irregular solutions to a turbulent financial market.

John Bantle, University at Buffalo

Detailed versus coarse models of possible breakdown of refuge strategy for agricultural pest management using pesticidal crops

Recent analytical and numerical studies of highly simplified models of pests in agricultural regions dominated by genetically modified pesticidal crops suggest that current government-mandated practices employing "refuges" of non-toxic plants may not be safe under some condition - promoting rather than delaying the proliferation of resistant pests. We explore this issue using a detailed stochastic model that accounts for pest life stages and behavior, farming practices, climate and weather, and resolves time at the 1-day level. We compare the predictions of the detailed and coarse models and attempt to explain the differences. Co-investigator: Ryan Klafehn

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Andrea Bongco, Kutztown University

An Apercus Study of Cardinality and Ordinality

We begin with a brief synopsis of where our work originated, the cardinality of N*, N, and subsets of them. We continue with a brief consideration of the origins of ordinality: partial orders on sets, linear orders, and the well ordering principle. We compare and contrast cardinality of a set with ordinality on the set. We present our argument of the Bongco claim, that given a well defined universe, U and a set, A, it is the case that $|A| < |P(A)|$. We conclude with a synopsis of other results.

Chelsea Cerini, Washington & Jefferson College

The Mystery of the M&m Sequences

In a 2005 CMJ article, Shultz and Shiflett introduced the idea of M&m sequences. Start with any three numbers x(1), x(2), and x(3). For $n \geq 4$, x(n) is defined to be the number such that the mean of (x(1),x(2),...x(n)) is equal to the median of (x(1),x(2),...x(n-1)). In the article, they proved that any M&m sequence can be transformed into a sequence beginning with 0, x, x +1 where $x \geq 1$. They showed that these sequences always stabilize with length 73 when $x \geq 21.3125$ and they conjectured that every M&m sequence stabilizes. In our research, we extend their result further and find new observations of our own. We also reveal the mystery behind the number 21.3125 and its significance with M&m sequences.

Eugene D. Cody, University of Kansas

Multivariate Vandermonde Determinants and Toric Codes

The minimum distance of toric codes has been studied extensively for various forms of polytopes. Furthermore, bounds for the minimum distance of toric codes for some polytopes P, subsets of m-dimensional real space R^m including the simplices of the form conv(0,le_1,...,le_m) have been determined using Vandermonde determinants. In our work, we derived lower and upper bounds to prove the exact minimum distance of some toric codes associated with the special polytopes P=conv(0,le_1,2le_2,3le_3), a subset of three-dimensional real space R^3.

Samantha Corvino, Slippery Rock University

Using Schwarz Triangles to Create Branched Packings on the Sphere

It has previously been discovered that copies of (2,3,5) Schwarz triangles containing arcs centered at two vertices create both a univalent and branched packing for a complex of 42 circles. In this talk, an extension of these principles will be presented by changing the arcs drawn within each triangle copy and by using different Schwarz triangles.

Joseph Crivelli, Vanderbilt University

A Delay Differential Equation Model for Cancer Virotherapy with Vesicular Stomatitis Virus

Despite recent efforts to develop new therapies for cancer treatment, few methods ever make it from the laboratory to the bedside. One promising form of treatment is oncolytic virotherapy. Oncolytic viruses preferentially infect and replicate in cancerous cells, leading to the destruction of tumor populations while sparing normal cells. We develop a mathematical model of virotherapy treatment with vesicular stomatitis virus (VSV), an oncolytic virus which selectively kills a wide range of human tumor cell types. VSV can infect cells during all parts of the cell cycle except during the resting phase. Since the interphase of the cell cycle has a minimum biological time course, we use delay differential equations to model the cell cycle-specific nature of the VSV system. We investigate how the length of the time delay and other parameter values affect the stability of the equilibria, and therefore, the efficacy of the treatment.

Amalia Culiuc, Mount Holyoke College

Energy of Graphs and Matrices

Let G be a finite, undirected, and simple graph. If {v1, …vn} is the set of vertices of G, then the adjacency matrix A(G) = [aij] is an n-by-n matrix where aij= 1 if vi and vj are adjacent and aij = 0 otherwise. The energy of a graph, E(G), is defined as the sum of the absolute values of eigenvalues of A(G). The concept of energy originated in chemistry and was first defined by I.Gutman in 1978. It has been generalized recently as follows: For a graph G on n vertices, let M be a matrix associated with G. Let l1, l2,…ln be the eigenvalues of M and let l be the average of those eigenvalues. The more general M-energy of G is then calculated as the sum of absolute deviations from the mean. In this talk we present our results on graph energy when M is the Laplacian matrix, the signless

Laplacian matrix, or the distance matrix. In particular we give bounds for energy of different graph classes and study the effect of edge deletion.

Jonathan Cummings, University of Nebraska, Lincoln

Signed Edge k-Domination Numbers of Graphs

The closed neighborhood N_G[e] of an edge e in a graph G is the set consisting of e and of all edges having an end-vertex in common with e. Let f be a function on E(G), the edge set of G, into the set {-1,1}. If the sum of all values of f(x) (over the set of x in N[e] is greater than or equal to k for each edge e in E(G), then f is called a signed edge k-domination function (SEkDF) of G. In this talk, I will define the signed edge k-domination number gamma'_{sk}(G) of G and will present a formula for gamma'_{sk}(G) for complete graphs and complete bipartite graphs. I will then, for any simple graph G, present the lower bound |V(G)| - |E(G)| + k – 1 for gamma'_{sk}(G), and characterize all graphs that achieve this lower bound. I will also present a specific upper bound for gamma'_{sk}(G).

John Dambra, University at Buffalo

The Dynamics of a Nephron

The nephron is the basic functional unit of the kidney. The rate at which blood enters the nephron is referred to as the glomerular filtration rate (GFR). GFR is controlled by two regulators called the tubuloglomerular feedback (TGF) system and the myogenic response. With both deterministic and stochastic numerical simulations, we have observed complex dynamics arising in the system. Moreover, using a mean reverting stochastic process, we model and study changes in parameters and system dynamics.

Wayne DeFloria, University of Pittsburgh at Greensburg

Symplectic and Contact Forms on Lie Groups

In this presentation, we will explain the definitions of Lie groups, Lie algebras and their matrix representations. As an application, we will show how to use the representations obtained for Lie groups to obtain symplectic and contact forms on Lie groups. If there is time, we will also show through examples how to use Maple to solve this problem.

Avis Foster, George Mason University

Numerical Modeling and Analysis of Fluid Structure Interaction in Biological Systems

This research work expands on an analytical solution for a mathematical model of blood flow through an artery, and the blood's interaction with the arterial wall and the surrounding cerebral spinal fluid. A mathematical model is developed using a wave equation and boundary conditions derived from a combination of Fourier series, a spring-mass equation, and a simplified Navier-Stokes Equation. The coupled system is solved via a method of lines numerical approach that predicts the mechanics of the arterial wall. The resultant model was validated against the analytical solution and analyzed for application to cerebral brain aneurysms. Influence of various model parameters is also investigated.

Nicki Gaswick, University of Nebraska, Lincoln

Sound Diagnostics: Guitar Chord Recognition

Sound diagnostics is a broad field encompassing applications such as distinguishing between animal calls to recognizing vocal pitches in popularized video games. Specifically our research hoped to show that a computer could differentiate between pitches to definitively recognize the sound of a failing machine. We modeled the sounds/pitches a machine makes using guitar chords. Creating a library of accepted machine sounds, we used Daubechies wavelets and Fourier analysis to create an algorithm that was able to compare new guitar chord samples to our accepted library. In successfully creating the algorithm that can differentiate between closely related cords, we determined that an algorithm may be created that could differentiate between pitches of failing machines and properly working machines.

Peter Gentile, Washington and Jefferson College

The Spaghetti Model: DNA Sequencing using Graph Theory

Graph theory is used to model a specific application of the Physical Mapping problem, which is to reconstruct the relative position of DNA fragments along the genome from pair wise overlap information. We call the model that we address in this talk the spaghetti model. A DNA strand is copied many times and cut into pieces of varying length using either an enzyme or a cleaving process, much like cracking handfuls of identical hard pieces of spaghetti. The pieces are then separated by length, and similar sized pieces are collected. Graph theory is used to order this collection of fragments. This problem can be mathematically modeled using a graph called a unit interval graph. An interval graph G is a graph whose vertices can be assigned to intervals on the real line such that two vertices are adjacent in G whenever their corresponding intervals overlap. A unit interval graph is an interval graph in which the intervals are of equal length. We prove that given a unit interval graph that models the overlap data, one can successfully order the pieces of the DNA sequence, and we give an algorithm to that effect. We also provide an algorithm which produces a minimum set of intervals necessary to order the entire DNA sequence.

Matt Grimes, Arizona State University

Transient growth effects in quasi-Keplerian Taylor-Couette flows

According to the Rayleigh criterion the hydrodynamic rotation profiles of Keplerian astrophysical flows are linearly stable. Thus, the onset of turbulence in this class of flows must arise due to other physical mechanisms, the most widely accepted candidate being the Magneto-Rotational-Instability. In addition, other mechanisms such as temperature and stratification gradients have been discussed and proposed in the literature as being capable to destabilize Keplerian flows. Here it is argued that even in the purely hydrodynamic case, the rotational profile may be subject to large temporary amplification of perturbations. We present a transient growth analysis of

incompressible flows with Keplerian profiles and show that substantial amplification, in the order of G 100, is achieved at Reynolds numbers of about 105. This amplification factor has been found to be enough to trigger subcritical transition to turbulence in a variety of shear flows. The numerical analysis is carried out on Taylor–Couette flows in the co-rotating Rayleigh stable regime using the infinite-cylinder approximation. This assumption avoids end-effects that may lead to hydrodynamic global instabilities. The most dangerous instability modes are found to be qualitatively different from those that emerge in plane Couette flow and also in counter-rotating Taylor–Couette regimes. In particular, the strong mean rotation in the system favors the amplification of perturbations that are invariant in the spanwise (axial) direction. A wide range of curvatures and Reynolds numbers are explored. The results will be discussed and interpreted in the light of recent experimental findings performed in an apparatus that is specifically designed to minimize end-effects in order to approximate the idealized case.

Rebecca Hager, SUNY Buffalo

Terrain and Geometry Effects in a Two-species Battle Model

We used a cellular automaton to model a two-species battle with intelligent attackers and random defenders. Once the simple battle model was sufficiently explored, we implemented terrain modifications. Cities are represented by areas of higher defense density in our model, and produce some interesting effects. Restrictions of the battlefield geometry were also explored. A large impassable central obstruction was added to the field; as the obstruction grows to be on the order of the field size, approximating a 1D system, the system shows qualities of jamming.

Robert Immendorf, Penn State University

Interesting Results about Dickson Polynomials

We present two proofs of results that eliminate certain classes of polynomials from being permutation polynomials. Our approach is to look at the effects of changing the alpha parameter in the classically defined Dickson polynomials of the first kind. The proofs use the Dickson polynomial recurrence and generating function manipulations.

Jonathan Jaquette, Swarthmore College

Existence of topological entropy preserving subsystems weakly embeddable in symbolic dynamical systems

The topological entropy of a dynamical system is known to be equal to or greater than all of its subsystems. We show conditions in which subsystems have equal topological entropy. Furthermore, we characterize systems containing invariant subsets of equal entropy that are weakly embeddable in a symbolic dynamical system. Positively expansive topological dynamical systems on compact metric spaces contain such subsets.

Heejoon Jo, SUNY Buffalo

Strategies for Delaying or Preventing the Development of Insect Resistance to Bt Cotton

We have constructed a computer model of the population dynamics of a primary insect pest of cotton. The model is fully stochastic, spatially explicit, and has fine time-resolution (one-day). It allows us to explore the effectiveness of various pest control strategies, in particular those that involve the use of genetically modified insecticidal crops.

Proliferation of a resistant strain of the pest is a significant danger to the long-term usefulness of such crops. Finding ways to prevent or delay this is of great economic importance. The model allows us to test strategies that make use of non-toxic "refuge" fields in static or rotated arrangements in combination with various planting schedules. We present the latest results of our simulations.

Benjamin Kanouse, Buffalo State College

Random Birefringence and its Impact on Optical Fiber Communications

Polarization mode dispersion (PMD) is one of the main obstacles facing future advancement of optical fiber communication systems. We explore different models of PMD generation in installed fiber transmission links and we quantify the PMD-induced outages in each of these models.

Abigail Kirchman, St. Olaf College

Classification of Loops of Generalized Bol-Moufang Type

A loop identity Bol-Moufang type if the same 3 variables appear on both sides of the equal sign in the same order, one of the variables appears twice on both sides and the remaining two variables appear once on both sides. One can generalize this definition by allowing different variable orders on either side of the identity, e.g. ((xx)y)z=x(y(xz)). There are 1215 nontrivial identities of this type. Loop varieties axiomatized by a single identity of this type are said to be of generalized Bol-Moufang type. We show that there are 48 such varieties: the 14 varieties of Bol-Moufang type, the 6 varieties of commutative Bol-Moufang type, and 28 new varieties.

Josh Koslosky, Duquesne University

Image Denoising Via Feature-Based Sparse and Redundant Dictionaries

In recent years the computer vision community has demonstrated that sparse and redundant representations of image patches can be used to denoise images. These representations can be formed using dictionaries that are either fixed (e.g. Discrete Cosine Transform) or learned from the noisy data itself. Finding the best patch representation leads to a constrained optimization problem, which depending on its formulation can be nonconvex. Elad and Aharon propose such a model which learns the dictionary from the noisy data, which they solve using Orthogonal Matching Pursuit and K-SVD (a modification of the Singular Value Decomposition inspired by K-means). In this talk we propose a modification of their algorithm in which dictionaries can be tailored to denoise smooth regions, textured regions, and edges separately. In particular, we discuss several approaches for segmenting an image based on these different geometric properties, and how dictionaries tailored to these properties can improve both the image representation and denoising.

Kun Justine Leng, SUNY Buffalo

Modeling Structural Change in Markets

Today's markets are increasingly dynamic and competitive. To be successful, firms have to be able to learn faster than and learn from their competitors. There is an ever growing need for modeling tools to help policy makers and management teams study a firm's choices and explore phases of industry evolutions. Traditional industry simulation captures dynamic behaviors of system components, only if the current structure remains stationary. To bridge the gap between behavior and structure, we are interested in examining how a single player's behavior can affect the overall structure of a complex system, by employing a modeling methodology called variable structure modeling, introduced by Oeren and Zeigler. Such models entail the possibility of changing the composition of the system as well as the interaction between its components, slightly different from the approach employed in evolutionary economics. Variable structure simulation treats each individual player in an industry as an agent, and investigates the agent's entry and exit strategies as well as the dynamics of his behavior. The consequent modeling framework more accurately describes a wide range of structural changes, as well as evolutions in individual behaviors. It may help us better understand dynamic industries and markets that possess endogenously-varying structures. This talk illustrates the features and power of this methodology and outlines an application to the U.S. housing market.

Steven Leuthe, SUNY Buffalo State College

Numerical Bifurcation Analysis in the Combined TGF + AA Model

The nephron is the basic functional unit of the kidney, filtering waste from the blood to be excreted as urine. The rate at which blood enters the nephron is referred to as the glomerular filtration rate. Glomerular filtration rate is controlled by two regulators, the tubuloglemerular feedback system in the distal portion of the ascending limb, and the myogenic response in the afferent arteriole. We numerically determine the locus of a Hopf bifurcation arising through the interaction of these two regulators, and demonstrate more complex, quasi-periodic dynamics.

Kym Louie, Harvey Mudd College

Statistical Modeling of Gang Violence in Los Angeles

Gang violence has plagued the Los Angeles policing district of Hollenbeck for over half a century. With sophisticated models, police may better understand and predict the region's frequent gang crimes. The purpose of this research is to model Hollenbeck's gang rivalries. A self-exciting point process called a Hawkes process is used to model rivalries over time. While this is shown to fit the data well, an agent based model is presented which is able to accurately simulate gang crimes not only temporally but also spatially. Random graphs generated by the agent model are compared to existing models developed to incorporate geography into random graphs.

Marc Mace, Abilene Christian University

Discrete Logarithm Problem over Composite Moduli

In an age of digital information, security is of utmost importance. Many encryption schemes, such as the Diffie-Hellman Key Agreement and RSA Cryptosystem, use a function which maps x to y by y=g^x (mod n) for a given n and a generator (or primitive root) g. The inverse of this function - trying to find x from y - is called the discrete logarithm problem. In most cases, n is a prime number. In some cases, however, n may be a composite number. In particular, we will look at when n=p^b for a prime p. We will show different techniques of obtaining graphs of this mapping and then we look to see whether the above mapping for the described n looks like a random map, and, if it does not, observe what we can that would help in solving the discrete logarithm problem.

Zachary Marzec, University at Buffalo

Efficient Methods to Compute Failure Probabilities in Optical Fiber Communications

Failures in many industrial systems are extremely rare by design and are consequently difficult to predict. We develop an adaptive variance reduction technique that combines Importance Sampling and the Cross Entropy Method, and we implement a parallelized version of it. Using these methods, we accurately model and predict the occurrence of birefringence-induced failures in installed optical fiber communication systems.

Brendan McVeigh, Swarthmore College

Dependence on initial conditions of evolutionary selection in spatial models

We have found that in a spatial model where infection transmission rate and infection duration are allowed to evolve independently we observe different evolutionary outcomes determined by initial conditions. In particular we observe that pathogen evolution results in either maximization of the infection duration or transmission rate. In previous works evolutionary selection for increased transmission rate was observed but not selection for infection duration.

Christian Miedel, Washington and Jefferson College

A vertex ordering result for an application of DNA sequencing using tripartite unit probe interval graphs

Graph Theory has been proven a useful tool in the Physical Mapping problem which is to reconstruct the relative position of DNA fragments along the genome from pair wise overlap information. Some cloning techniques generate clones that are approximately the same length, and their overlap information can be modeled using unit probe interval graphs. A graph is a probe interval graph if its vertices can be partitioned into two sets, probes and nonprobes, with an interval assigned to each vertex so that vertices are adjacent if and only if their corresponding intervals intersect and at least one of the vertices is a probe. A unit probe interval graph has intervals of the same length. If one assumes that we start with a contiguous piece of DNA, then we can assume a canonical order exists on the vertices reflecting the real order of fragments. We investigate the situation in which we have three restriction enzymes cutting DNA into fragments of approximately the same size. We prove that with the exception of vertices occurring at the ends of the interval model, our resulting tripartite unit probe interval graph has a realizable consecutive ordering of the vertices if and only if it is connected, which would provide the ordering of the DNA fragments.

Michael Mikucki, Colorado State University

Performing Sensitivity Analysis for Ecological Models

Sensitivity analysis is a critical tool when interpreting the solutions to discrete stage structured mathematical models in ecology. The analysis informs the researcher which components of the model are most significant when considering a given output of the model, and by relating the precision of model outputs to the precision of the inputs, indicates which parameters of the model must be estimated most accurately. Sensitivity analysis has major implications for management strategies as well as for experimental design. However, as ecological models become increasingly sophisticated, sensitivity analysis becomes increasingly difficult and time consuming to perform. This presentation will describe the basic functionality and implementation of a graphical user interface which, when provided with a discrete time nonlinear stage-structured model, performs the sensitivity analysis automatically without requiring additional input. The user may select an adjoint based approach to provide sensitivities with respect to given quantities of interest at reduced cost. This approach is easily generalized to continuous time stage-structured models based on ordinary differential equations.

This work was supported by the National Science Foundation through the Interdisciplinary Training for Undergraduates in Biological and Mathematical Sciences (UBM) program.

Jorge Wu Mok, SUNY Buffalo

Methods for victory in a two-species battle model

To explore the question of how to win a battle, we created a simple model: two opposing species on a lattice, one intelligent and one random. Initial results showed that, contrary to our expectation, having too much information causes the intelligent force to lose more easily. To explore the causes of this, we used two methods. Analysis of the initial conditions allowed us to find regions of the parameter space that are favorable to the attackers. Also, we tried various modifications to the strategy of the intelligent force; an adaptive strategy was shown to be particularly effective.

Sylvia Naples, Smith College

Adinkra Graphs: Pictures of Supersymmetry

The laws of physics are symmetric; transformations in space and time leave the laws unchanged. Supersymmetry is a symmetrical theory of fundamental particles: bosons and fermions. A supersymmetric transformation exchanges boson and fermion superpartners with the same mass and charge, and likewise leaves the physics unchanged. Much research in supersymmetry is motivated by an interest in String theory, a theory that may unify the fundamental forces of nature, and one that requires supersymmetry if it is to successfully model the known universe.

In 2004, physicist Jim Gates developed Adinkra graphs as a new approach to a 30 year old problem: to classify all possible supersymmetric pairings in N dimensions. The supersymmetric pairings are typically represented in a supersymmetry algebra by a slew of lengthy equations. An Adinkra is an N-regular, bipartite graph with several additional properties that contains all of the information in these formulas. The Adinkra is a visual representation of the algebra, and permits the use of graph theoretical tools. We define homology on Adinkra graphs and prove an interesting result: the Betti numbers for an Adinkra are given by a polynomial of degree less than N. Additionally, we provide a counterexample to such a classification that suggests the key to a complete classification.

Cristian Niculas, Gainesville State College

Mathematical Modeling of Delamination Growth under Gas Pressure

Delamination is defined as the separation of the surface layer from the solid body of pipe line. A perfect example of this process is the hydrogen-induced delamination. Hydrogen-induced delamination occurs when hydrogen is transported to a certain location, through a pipe. The metal in the pipe absorbs hydrogen gas into its small cracks, which are a result of manufacturing imperfections. As the hydrogen is absorbed into these cracks, it reacts with other hydrogen atoms and forms the H2 molecules. Throughout this process, the pressure inside the crack increases and consequently, the crack begins to grow. Studying hydrogen-induced delamination growth is important because the corrosion can determine the sections of pipe which may be in danger of rupturing and the intervals at which certain pipes must be replaced. Such an understanding can prevent disasters that would otherwise damage the environment. While researching hydrogen-induced delamination growth, the first, the delamination (crack) opening should be obtained. Next, the volume of the crack can be derived by integrating the crack opening over the crack area. The expression for the volume is then substituted into the Ideal Gas Equation, resulting in the integral equation for the radius of the delamination. The integral equation is then reduced to the differential equation using the Fundamental Theorem of Calculus. After solving the equation, the analytical expression for dependence of the radius of the delamination on time is derived.

Katherine Osenbach, The University of Scranton

Electrical Impedance Imaging of Corrosion on a Partially Accessible 2-Dimensional Region

The inverse problem of determining the amount of corrosion on an inaccessible surface of a two-dimensional region is examined. Using numerical methods, we develop an algorithm for approximating corrosion a profile using measurements of electrical potential along the accessible portion of the region. We also evaluate the e□ect of error on the problem, address the issue of ill-posedness, and develop a method of regularization to correct for this error. An examination of solution uniqueness is also presented.

Marlene Ouayoro, George Mason University

Retrieving Economic Parameters in Asset Flow Equations

This research attempts to replicate the observed price points of financial instruments by modeling with the Caginalp-Balenovich differential equation. We fit the equation to the observed curve using the Gauss-Newton Method to estimate the initial parameters and initial values of the data.

Benjamin Pearce, Slippery Rock University

Deal or No Deal?

The purpose of this talk is to find out how the banker in the game show "Deal or No Deal?" calculating his offer. I will compare the offers he makes to the expected value of the remaining amounts and I will talk about how the offers accepted compare to the expected value of the remaining amounts.

Tobit Raff, Whittier College

Computations with Erdos' minimum modulus problem

One of Paul Erdos' favorite open problems asks if it is possible to cover the integers with a finite set of linear congruences having distinct moduli greater than M, for M arbitrarily large. We survey methods used to study this problem, paying particular attention to computational techniques. We establish a brute force attack and a greedy algorithm balancing complexity and guaranteed results for small M. We consider desirable characteristics for an "optimal" covering and construct algorithms that determine whether a covering system with a given minimum modulus is most efficient.

Zachary Robinson, Penn State University

The Pressure Gradient System

The pressure gradient system is a sub-system of the compressible Euler system. It can be obtained either through a flux splitting or an asymptotic expansion. In both derivations, the velocity field is treated as a small remnant of the original velocity of the Euler. As such, the boundary conditions for the velocity do not necessarily follow the original ones and careful consideration is needed. We provide numerical simulations as well as basic characteristic analysis and physical considerations for Riemann problems to the pressure gradient system to find out appropriate boundary conditions, including internal conditions at the origin. The study reveals subtle structures of the velocity: Both components vanish at the origin with possible square root type singularity. Comparing to the roll-up of shear waves or vortex-sheets of the Euler system, these mild singularities are forced by the system to occur only along rays from the origin, which indicates a desirable trait of the model. The numerics is done via the automated clawpack, for which we provide the Riemann solvers in both the normal and transversal directions, where the Roe's approximation has the elegant 1/2 average in the system's original variables.

Cynthia Rush, University of North Carolina at Chapel Hill

Extracting Biological Information from Networks Representations of Genetic Interaction

Genetics, like many biological disciplines, has only just recently begun to be studied by mathematicians. Often computational geneticists wish to extract the maximal amount of biological information from genetic interaction datasets. This can be done by maximizing the context-dependent complexity measure of their corresponding networks representation [1]. Here, I looked at various networks structures to find which architecture best encodes this complexity, ultimately concluding that modular networks naturally store biological information more efficiently than a random or scale-free structure. This goes against the classic 'pathway' structure of genes by suggesting that, rather, communities of genes interact in distinct and prescribed ways.

[1] Carter GW, Galas DJ, Galitski T (2009) Maximal Extraction of Biological Information from Genetic Interaction Data. PLoS Comput Biol 5(4): e1000347.doi:10.1371/journal.pcbi.1000347

Amanda Sgroi, Duquesne University

Texture Analysis as a Technique in Support of Image Denoising

Texture analysis has become a widely explored problem in image processing due to its vast applications, for instance, image denoising, inpainting, and object recognition. In the case of image denoising, texture analysis can be used to more accurately remove noise depending on the varying complexity within an image. In this talk I will discuss two algorithms we have developed that use texture analysis in this way. The first is a perceptually adaptive version of bilateral filtering, which uses the variations in texture to separate regions, analyze the distinctions between textures contained in various regions, and remove noise throughout the image based on these findings. This method also uses Daly's Visual Difference Predictor which visualizes the spatial frequencies processed by the amplitude-nonlinearity function in the frequency domain, in relation to various orientation decompositions. The second algorithm, which denoises using sparse dictionary representations, uses tools such as entropy, canny edge detection with human texture analysis input, and a combination of morphological operations in conjunction with statistical information for texture analysis to separate the image for the multiple dictionaries. Although these techniques vary in application, their implementations contain many similarities, which can be applied to other areas in image processing.

Robert Short, John Carroll University

Taking Flatland to the Limit

We'll take a look at the geometry within Flatland - the book by Edwin Abbot. Then we'll see what happens as we use calculus to take these geometric concepts to their limits.

Sommer Sprowls, Washington & Jefferson College

Set Differences and Jump Sequences

The study of set differences dates back to the 1940's by prominent mathematicians, including P. Erdös, P. Turán, John Leech, J.C.P. Miller, and B. Wichmann, to name a few. Let A be a finite set. Define the set difference $A - A = \{a - a | a \in A\}$. Let $n \in \mathbb{N}$. Denote $[n] = \{0,1,2,..., n\}$. Let $A \subseteq [n]$ be a k-element subset, and denote the cardinality of A by |A|. A is said to be n-complete if $| A - A | = | [n] - [n] |$. That is, the set A-A generates all possible 2n +1 differences in [n]-[n]. In our studies, we tried to find methods to create the smallest n-complete subsets of [n], called bases. We examined processes already investigated by others, such as the modular arithmetic method and the method developed by Wichmann and improved by Miller. We also explored jump sequences, which are created by the differences between values in A, and we compiled a list of requirements needed for these sequences to result in complete sets. Our paper combines and details the many different approaches that can be used to create n-complete sets, most of which have been scattered about in various places for over sixty years.

Jamahl Stokes, University of Maryland at Baltimore County

A Study of Water Waves

Herein we conduct laboratory experiments on surface water waves generated by either a mechanical wave maker or by wind. We consider dispersion and damping of monochromatic wavefields, and the generation and subsequent evolution of wind-generated waves. Measurements of frequency and wavelengths agree well with predictions from linear theory. Measured damping rates are reasonably predicted by a model that assumes an inextensible surface. For a fixed wind source, we were able to mathematically characterize the wind field from measurements. Close to the wind the wavefield was broad-banded. Further downstream the spectrum developed a peak that downshifted with increasing distance from the wind source. The energy of the wavefield grew approximately exponentially in distance until it peaked and decayed approximately exponentially with distance. Both the location of the transition between growth and decay and the subsequent decay rate decreased with total duration of the wind. When we generated mechanical waves in addition to the wind waves, the spectrum behaved similarly, although the peak seemed to form first near the frequency of the mechanically generated waves and ultimately downshifted to a lower frequency. The energy of the wavefield followed the same trend, although it was larger throughout; it peaked sooner; and it decayed at a larger rate.

Kaylee Sutton, John Carroll University

Pell's Equation and quadratic non-residues

The equation y^2 - nx^2 = +/- 1 is Pell's Equation. When n is an odd prime, the theory of quadratic resides and non-residues applies or can give insights to solutions.

Reid Thornton, Colorado State University

Whitebark pine (Pinus albicaulis) density-dependent blister rust infection model

Whitebark pine (Pinus albicaulis) is a high-elevation, five-needle pine tree species located in the northwestern United States and Canada. In the early 1900s, blister rust (Cronartium ribicola), an invasive fungus, was introduced to the northwestern United States and is now spreading southeast along the northern Rockies. We developed a discrete-time, stage-structured model that projects the evolution of a sample one-hectare population in the face of infection. We examine the crucial role of population density and the effect of a rare allele in the Whitebark pine population that confers resistance to blister rust. We performed an elasticity analysis with respect to both the model parameters and initial conditions in order to identify which must be estimated with greatest accuracy and to suggest possible future management strategies.

Christopher M. P. Tomaszewski, Villanova University

A Spectral Analysis of Cyclic and Elementary Abelian Subgroup Lattices

We consider the classical subgroup lattice of a group as an unlabeled, undirected simple graph and investigate the spectrum of its associated adjacency matrix. In so doing, the spectral radius — which allows an approximation, for sufficiently large n, of the number of n-walks in the graph between any two vertices — is found. The spectra for all graphs derived from cyclic groups are determined in general, as well as those derived from certain families of small elementary Abelian groups.

Melissa Wasilewski, University of Scranton

Network Models for Influenza Spread and Interventions in Schools

Person-person contacts in schools heavily influence disease dynamics and epidemic sizes for society at large in the case of diseases such as influenza. For this reason, and also to assess possible school interventions, one would like to better understand the structure of the associated student contact networks. We have developed a methodology to construct highly accurate contact networks, and applied this to data from a Virginia high school. The constructed networks were characterized in terms of graph measures such as degree distribution and cluster coefficients as well individual-based SEIR disease spread for a flu-like disease. With our approach, certain interventions were addressed in a direct manner, and we studied their effects using graph measures and SEIR dynamics. The tested interventions lowered the average degree of the contact graphs and decreased the severity of the epidemic. The epidemic peak occurred later than it had without interventions, which could potentially give students more time to obtain vaccinations that would further lessen the severity of the disease.

Adam Yusko, Lehigh University

Representing Split Graphs as Probe Interval Graphs

Probe interval graphs are interval graphs with its vertices partitioned into probes and non-probes. There is an edge between two vertices if the intervals representing them have a nonempty intersection, and at least one of the vertices is a probe. An important step towards understanding when a graph is also a probe interval graph involves understanding when a split graph can be represented as a probe interval graph.

Hao Zou, Macalester College

On Determining the Number of Clusters - An Empirical Study of Different Algorithms

In this project, we perform one of the first empirical tests comparing several existing algorithms for determining the number of clusters in a data set (the gap statistic, X-means, G-means, data spectroscopic clustering and self-tuning spectral clustering). We use a large number of data sets randomly generated with varying distributions (normal and uniform distributions) and parameters (dimensions, number of clusters, number of data points per cluster, and degree of separation between points). The results show that the G-means and X-means perform best on the majority of test cases. In addition, the gap statistic returns good estimates for smaller dimensions and number of clusters, but is less accurate and much slower when they get bigger. So, we explore ways to improve the gap statistic, and formulate the problem in the simplified continuous context to consider its theoretical basis.